Problem Set 1. Due Thursday, February 9.

1. Suppose \( V(x, y, z) \) is a solution to Laplace’s equation in the half space \( z \geq 0 \). Suppose

\[
V(x, y, 0) = \begin{cases} 
1 & \text{for } 0 \leq x \leq a \\
-1 & \text{for } -a \leq x < 0 \\
0 & \text{for } |x| > a 
\end{cases}
\]  

and that \( V(z = \infty) = 0 \). Find an expression for \( V(x, y, z) \) that is valid everywhere in the half space. Find (or look up) analytical expressions for all integrals. (You can use integral tables; for example, Gradshteyn and Ryzhik in the Engineering-Math-Physics Library)

2. Suppose the earth has a spherical surface with radius \( r = a \). Suppose \( \partial_r V = p + q\sin^2\theta \) at \( r = a \), where \( V \) is the earth’s gravitational potential and the constants \( p \) and \( q \) are known. Find \( V(r, \theta, \phi) \) for all \( r > a \).

3. It is likely that the boundary between the outer core and the mantle has topographic features. Suppose the typical wavelength of this topography is 1000 km, and the peak-to-trough amplitude is \( h \). Assume \( h \ll a \) (\( a \) is the earth’s radius). Estimate the amplitude of the gravity signal at the earth’s surface caused by this topography, as a function of \( h \). How large would \( h \) need to be to produce a 10 \( \mu \text{gal} \) effect on surface gravity? Assume the density of the outer core is 11 gm/cm\(^3\), the density of the lower mantle is 5 gm/cm\(^3\), and the average radius of the core-mantle boundary is 3480 km.