Blackbody Radiation and the Quantization of Energy

Energy quantization, the noncontinuous nature of energy, was discovered from observing radiant energy from blackbodies. A blackbody absorbs electromagnetic (EM) energy efficiently. An ideal blackbody absorbs all electromagnetic energy that bears upon it. Efficient absorbers of energy also are efficient emitters or else they would heat up without limit. Energy radiated from blackbodies was studied because the characteristics of the radiation are independent of the material constitution of the blackbody. Therefore, conclusions reached by the study of energy radiated from blackbodies do not require qualifications related to the material studied.

If you heat up a good absorber (emitter) of EM energy, it will radiate energy with total power per unit area proportional to the fourth power of temperature in degrees Kelvin. This is the Stefan-Boltzmann law:

\[ R = \sigma T^4, \]

where the Stefan constant \( \sigma = 5.6703 \times 10^{-8} \text{ W/m}^2\text{K}^4 \). This law was first proposed by Josef Stefan in 1879 and studied theoretically by Boltzmann a few years later, so it is named after both of them.

The EM energy emitted from a blackbody actually depends strongly on the wavelength, \( \lambda \), of the energy. Thus, radiant power is a function of wavelength, \( R(\lambda) \), and total power per unit area is simply an integral over all wavelengths:

\[ R = \int R(\lambda) d\lambda. \]  

(1)

The function \( R(\lambda) \) is called the blackbody spectrum. Recall that

\[ c = f\lambda = \frac{\omega}{k}, \quad \omega = 2\pi f, \quad k = \frac{2\pi}{\lambda}. \]

The blackbody spectrum is different for different temperatures, but has the same general shape as the solid lines in Figure 1 show. More total power is emitted as the temperature of the blackbody grows and the peak of the spectrum, \( \lambda_m \), shifts to shorter wavelengths. In fact, \( \lambda_m \) is inversely proportional to temperature \( T \) such that:

\[ \lambda_m T = \text{constant} = 2.898 \times 10^{-3} \text{mK}, \]

which is known as Wiens’ displacement law because it quantified the displacement of the spectral peak as temperature grows.

Without going into any detail, it turns out that classical EM theory predicts that

\[ R(\lambda) = 2\pi ckT\lambda^{-4} \quad \text{Rayleigh-Jeans Law}, \]  

(2)

where \( k = 1.381 \times 10^{-23} \text{ J/K} = 8.617 \times 10^{-5} \text{ eV/K} \) is the Boltzmann constant. Note that at room temperature the quantity \( kT \approx 1/40 \text{ eV} \). Inspection of this prediction, shown in Figure 1 as the dotted line, indicates that it does not agree with observations of the blackbody spectrum. In fact, not only is there disagreement, but substitution of equation (2) into (1) results in the catastrophic result that the power per unit area radiated by a blackbody is infinite. Because the disagreement between the observed curves and the curve predicted by the Rayleigh-Jeans formula worsens as period decreases this is called the ultraviolet catastrophe. Recall that ultraviolet light is shorter wavelength than visible light which is itself shorter wavelength than infrared, etc.
Figure 1: Solid curves are blackbody spectra predicted by Planck’s Law (eqn (4)) for the three temperatures indicated, and the dotted curve is the prediction from the Rayleigh-Jeans Law (eqn 2)) for a temperature of 1650 K.

One of the key assumptions in the derivation of the erroneous Rayleigh-Jeans law is the continuous nature of EM energy. Planck was able to model the shape of the blackbody spectrum much more accurately by assuming that energy is quantized, that it is not continuous:

\[ E_n = n hf \quad n = 0, 1, 2, 3, \ldots \]  

where \( h \) is Planck’s constant. With this assumption, in 1900 Planck rederived the theoretical shape of the blackbody spectrum and found the following:

\[ R(\lambda) = \frac{2\pi c^2 h \lambda^{-5}}{e^{hc/\lambda kT} - 1} \quad \text{Planck’s Law} \]  

Fitting this functional form, which is shown for three temperatures in Figure 1, for the unknown \( h \), Planck estimated Planck’s constant, current values of which are \( h = 6.626 \times 10^{-34} \) J-s = 4.136 \times 10^{-15} \) eV-s.

This is generally considered the earliest indication of the quantization of energy. Equation (3) is one that we will use throughout our study of quantum mechanics. It will turn out that energy is quantized by this formula for all forms of EM energy and for matter, as well.