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Size of Popocatepetl volcano explosions (1997-2001) from waveform inversion

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Abstract. Several volcanic explosions have been recorded since April 1997 at broadband seismic stations located around the Popocatepetl volcano, Mexico. We have inverted waveforms of ten of these explosions to estimate the following source parameters: depth, duration, magnitude and direction of the single force, F. The crustal structure used in generating Green's function at nearest stations is derived from the inversion of teleseismic receiver functions at the broadband permanent station PPIG, located 5 km north of the volcano. This inversion reveals a low velocity zone at ~8 km beneath the summit with high Poisson ratio, possibly related to the magma chamber. We find that F scales with τ, the duration of the source-time function, as $F \propto \tau^2$. Based on this relationship we determine an impulse magnitude scale, M_k. This magnitude is tied to the Mount Saint Helens initial explosive phase of May 18, 1980, whose magnitude is estimated as 4.6. M_k of the ten Popocatepetl explosions ranges between 1.8 and 3.2. Finally, we also propose an equivalent formula for rapid estimation of magnitude of future Popocatepetl explosions, which requires filtered amplitudes at PPIG.

Introduction

Popocatepetl is one of the most active volcanoes of Mexico. It poses significant hazard to population centers in its neighborhood. In fact, there are several million people who live within 60 km of its summit. Since 1993, the volcano has shown renewed activity, presenting an increase seismicity followed by large fumarolic and ash emissions. In April 1997, an explosive phase started destroying the lava dome, which had previously formed at the bottom of the crater. Since then the dome emplacement-destruction process has repeated itself several times [*Arciniega-Ceballos et al.*, 1999].

It is important to know source parameters of the explosions, especially their sizes, and their possible relationship with precursory activity. Towards this goal, we estimate source parameters of ten explosions that were recorded by broadband seismographs located around the volcano (Figure 1), assuming a single force model [*Kanamori and Given*, 1983]. We then propose a magnitude scale, M_k, based on impulse, K. M_k is tied to the magnitude of the initial phase of the Mount Saint Helens explosion of May 18, 1980 which is based on the Volcanic Explosivity Index (VEI) [*Pyle*, 2000] and which we estimate as 4.6. We finally derive an equivalent relation, which will permit a quick estimation of the magnitude of future Popocatepetl explosions from filtered wave-amplitude at the neighboring permanent station PPIG.

Crustal Structure

To determine the source parameters of the explosions at the nearest stations (PPIG, SXPP, PPC and SPP; Figure 1), we determined the crustal structure below the volcano applying a simulated annealing optimization algorithm [*Kirkpatrick et al.*, 1983] to invert receiver functions [*Langston*, 1979]. We use four teleseismic events from South America, recorded at the broadband station of PPIG located 5 km north of the volcano summit (Figure 1). Receiver functions were obtained from a time-domain deconvolution [*Ligorria and Ammon*, 1999] of the vertical P wave component from the corresponding radial component, using a gaussian filter with a high-frequency cutoff at 0.5 Hz. Very strong negative phases were found around 4 and 9 s after the first arrival (Figure 2a). Synthetic tests led us to include a shallow low-velocity zone (LVZ) to reproduce these features. Synthetic receiver functions showed extremely sensitive constructive and destructive interference patterns, which depended mostly on the thickness of the superficial layers. This behavior of the waveform provides a good constrain on the depth of the top of the LVZ.

The observed receiver functions were stacked to reduce noise. This procedure also allowed us to obtain a standard deviation band around the average of the stacked functions

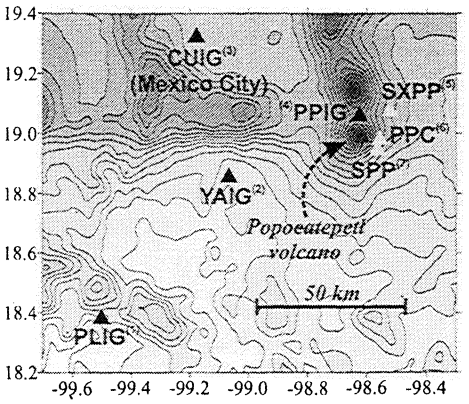

Figure 1. Location map of Popocatepetl volcano, showing topography and broadband seismic stations. Solid triangles: permanent stations; open triangles: portable stations.

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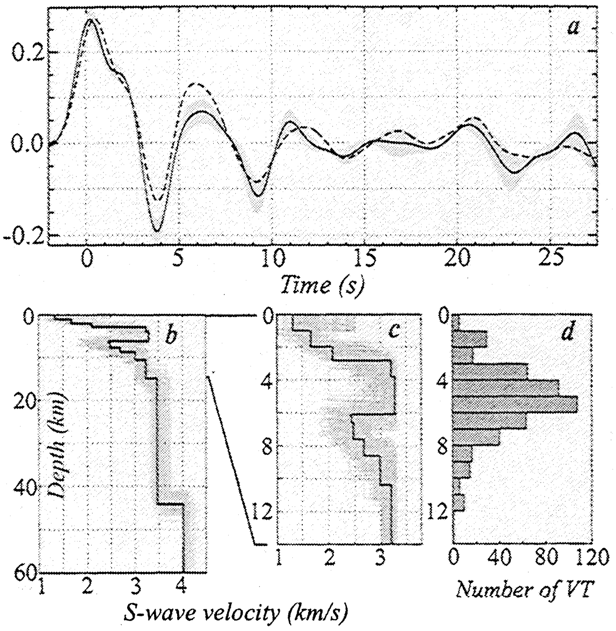

Figure 2. (a) Solid line: stacked receiver function. Dashed line: synthetic from best model. Shaded area straddling stacked receiver functions indicates one standard deviation. (b) Solid line: best crustal model from global inversion. Thin lines: 489 models which response lies almost within the data uncertainty. (c) Enlargement of shallow part of crustal structure from (b). (d) Depth distribution of 474 volcano-tectonic (VT) events from December 1994 to August 1999.

(Figure 2a) and, hence, an estimation of the uncertainty in the data. This error band was used in the simulated annealing inversion algorithm to select those models whose solutions (Figure 2b) lie as much as possible within the data uncertainty [*Cruz-Atienza*, 2000]. The inversion was performed for thickness and wave velocity at each layer, and the Poisson ratio, ν, of the LVZ. For the other layers, ν was fixed to 0.27 following previous estimations by *Cruz-Atienza* [2000].

The initial model was taken as the one obtained by *Cruz-Atienza* [2000] for station CUIG (60 km to the northwest of the volcano). The global inversion at PPIG station yields the expected LVZ, as well as two velocity gradients, well constrained, at the top of the structure and between 6 and 10 km (Figures 2b, 2c). The depth of the LVZ, around 6 km below the station (~8 km from the volcano summit), is similar to estimations of magma chamber at Long Valley Caldera [*Ponko and Sanders*, 1994]. Furthermore, a high Poisson ratio obtained for the LVZ ($\nu = 0.295 \pm 0.013$) and the average low Q_s value (~60) beneath the Popocatepetl volcano [*Shapiro et al.*, 2000], suggest the presence of high temperatures and partially melted rocks. The depth distribution of 474 VT best located events by *Valdés-González et al.* [2001] shows a large concentration of events above the LVZ (Figure 2d). *Lomax et al.* [2000] reported similar results for the Vesuvius.

The crustal structure used in the next section to compute Green's functions for the farther stations (CUIG, YAIG and PLIG), was taken from the previous teleseismic receiver function analysis [*Cruz-Atienza*, 2000] at CUIG (Figure 1).

Source Parameters of the Explosions

Four permanent seismic broadband stations of the National Seismological Service (PPIG, YAIG, CUIG and PLIG) and three portable seismographs (SXPP, PPC, SPP) recorded Popocatepetl explosions (Figure 1).

Following *Kanamori and Given* [1982, 1983] we model volcanic explosive sources in the far field as a single force applied to the ground. We neglect the isotropic contribution [*Kanamori et al.*, 1984] and assume a triangular source-time function for the force. Because of the linear relationship between the three components of the force and the associated components of the ground motion, the amplitude of the force components (F_x,F_y,F_z) are uniquely determined. Thus, it is possible to formulate a linear over-determined least-square inversion to obtain the best three-force components, provided that, the depth and source duration are known. Source depth and duration for the linear inversion are supplied by a grid search. The best solution is given by those values that minimize the least-square error between observed and synthetic seismograms. Thus, our inversion scheme provides magnitude, direction, depth, and duration of the applied force. The algorithm uses the discrete wave-number integration method coded by *Herrmann* [1996], to compute the complete wavefield due to an arbitrarily oriented single-force located along the volcano conduit. Solutions are not sensitive to the crustal structures shown in Figure 2b; the solid line indicates the model used in our calculations.

The data set consist of 27 explosions. The ten explosions for which the origin time could be determined unambiguously were inverted to obtain source parameters (Table 1). Prior to the inversions, the seismograms were rotated into radial and

Table 1. Popocatepetl explosions: Apr 1997 to Jan 2001

Date	*Time*	*Depth (m)*	τ (s)	(F_x,F_y,F_z) 10^9 N	*Stations*	M_k
*970429	06:12:01	200	9.6	(40, 24, 125)	1,3,4,6,7	3.2
*970514	03:31:47	400	6.0	(21, 6, 62)	1,2,3,4	2.8
*970514	14:50:17	600	6.0	(21, -3, 78)	1,2,4	2.9
*971225	01:29:12	0	8.0	(47, -20, 125)	1,2,3,4,5,6	3.1
*980102	00:27:27	100	5.8	(15, -1, 55)	2,3,4,5,6	2.8
*980921	16:47:60	200	5.8	(10, 0, 47)	1,2,3,4	2.7
980921	20:43:56				4	2.0
*980922	17:25:05	300	3.0	(6, -5, 21)	1,2,4	2.3
980923	23:29:30				4	2.5
981006	04:12:36				4	2.5
*981125	14:03:28	900	3.4	(8, 3, 13)	2,4	2.2
*981125	18:05:41	400	3.0	(6, -2, 10)	2,4	2.1
981125	22:58:38				4	2.7
981126	16:13:30				4	2.1
981127	03:13:40				4	2.4
981127	04:20:21				4	2.9
981128	05:45:02				4	2.5
981128	08:41:57				4	2.3
981129	09:05:50				4	2.6
981207	12:23:06				4	2.3
*981215	23:50:09	400	4.0	(5, 1, 28)	4	2.5
990320	00:38:37				4	3.0
990322	17:44:10				4	2.8
990404	08:25:49				4	2.5
990415	15:56:52				4	1.8
001217	08:36:56				4	2.4
010129	17:02:21				4	2.1

* Explosions whose source parameters were obtained from waveform inversion. F_x eastward, F_y northward, F_z downward. Station numbers are keyed to their names as shown in Figure 1.

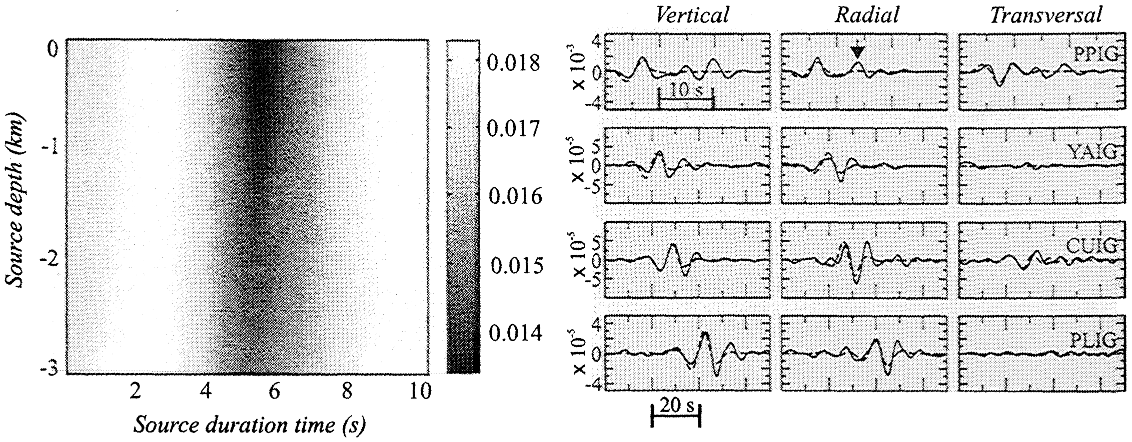

Figure 3. Inversion of May 14, 1997 (03:31:47) explosion. Left: L2 misfit function values for all combinations of source depth and duration during grid search. Right: observed (solid lines) and synthetics (dashed lines) seismograms.

transverse components, and re-sampled at 10 Hz. Near-source seismograms (from PPIG, PPC, SPP and SXPP) were bandpass-filtered between 5 to 30 s, while seismograms recorded by farther stations (YAIG, CUIG and PLIG) were filtered between 10 and 30 s. Figure 3 shows the result of the inversion of an explosion that occurred on May 14, 1997 (03:31:47). The left hand side of the figure indicates the L2-norm misfit values between observed and synthetic data for the entire grid search domain. Although a minimum occurs at 400 m and 6.0 s, it is clear that resolution of the source duration is higher than of the source depth. The optimal three components of the source are listed in Table 1. The right hand side of Figure 3 compares observed and synthetic seismograms. A large pulse seen ~5 s after the first arrival at the closest station (small arrow in Figure 3) corresponds to the air shock wave. For most events, transverse component seismograms show large amplitudes, implying that horizontal component of the forces is as significant as the vertical one.

Scaling Law and Magnitude

Force, F, plotted as a function of source duration, τ, in Figure 4a, clearly shows a $F \propto \tau^2$ scaling. A regression leads to the following relation: $\log F = 2.0 \log \tau + (9.24 \pm 0.1)$. The $F \propto \tau^2$ scaling is in agreement with theoretical expectations and observations [*Nishimura and Hamaguchi*, 1993]. On the other hand, from linear theory of elasticity we expect $F \propto A$ [*Aki and Richards*, 1980], where A is the wave amplitude. We define A as $A = sqrt(A_N^2 + A_E^2 + A_Z^2)$ (where A_N, A_E and A_Z are the peak amplitudes on N, E, Z components, respectively) and is determined from the bandpass-filtered seismograms (10 to 30 s) recorded at PPIG. From Figure 4b, the relationship is: $\log F = \log A + (14.07 \pm 0.08)$, where A is in cm/s and F is in N. The magnitude of an explosion may be defined by:

$$M = \log A + C \tag{1}$$

where C depends on distance, attenuation, and site effects. For a volcanic explosion whose source-time function can be approximated by a triangle, the impulse, K, is given by: $K = \tau F/2$. Since $\tau \propto F^{1/2}$ and $F \propto A$, it follows that $K \propto A^{3/2}$. Thus, equation 1 can be rewritten as $M = (2/3)\log K + C_1$, where C_1 is now a constant. We determine C_1 from the following considerations. The impulse of the first four subevents of the May 18, Mount St. Helens explosion, determined from the results of *Kanamori et al.* [1984], is 9.2×10^{13} N·s. On the other hand, using the magnitude relationship, $M = \log(mass) - 7.0$, equivalent to the VEI [*Pyle*, 2000], for a mass discharge rate of 4.0×10^9 kg/s during the first 110 s of St. Helens explosion (mean value of 2 to 6×10^9 kg/s given by *Brodsky et al.* [1999]), we get $M = 4.6$. This estimation has an uncertainty of ± 0.3. To estimate the constant C_1, we take the values of impulse and magnitude mentioned above. These considerations, along with proper accounting of the constants, lead us to a magnitude scale based on impulse, which has a general validity, defined by:

$$M_k = (2/3)\log K - 4.71 \tag{2}$$

We now define an equivalent magnitude scale that depends on the wave amplitude recorded at PPIG. In this case $M = \log A + C_2$, where C_2 is a constant. We determine C_2 by combining $K = \tau F/2$ with the relationship between log F and log τ, and log F and log A (given above and in Figure 4), and equation 2. The relationship is

$$M_k = \log A + 6.08 \tag{3}$$

where A is in cm/s. Since this magnitude scale depends on the wave amplitude at PPIG alone, and the data from this station is available in near real-time, it permits a very rapid estimation of the size of a volcanic explosion at Popocatepetl. It is important to note that M_k is fixed only at one point to the scale based on mass, and hence the two magnitudes will deviate from each other for larger and smaller explosions. Table 1 lists

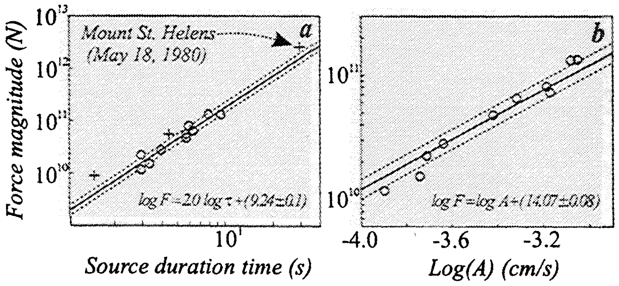

Figure 4. (a) Force, F, as a function of source duration, τ, for the ten Popocatepetl explosions (circles). Crosses: data from taken from Nishimura and Hamaguchi [1993]. (b) Relation between force and wave amplitude at PPIG station (circles). Solid lines show the best-fit curves, dashed lines are one standard deviation.

magnitudes, using equation 2, for ten explosions whose waveforms were inverted. For the remaining events, magnitudes were estimated from equation 3.

Discussion and Conclusions

Popocatepetl explosions since April 1997 can be modeled as point forces, which range between 1.17 x 10^{10} N and 1.35 x 10^{11} N. The horizontal components of these forces (F_x and F_y), on average, are $37 \pm 15\%$ of the vertical component. In general, forces point towards the east (see Table 1).

The force, F, scales with the duration, τ, as $F \propto \tau^2$, in accordance with theoretical predictions. Based on this scaling, the relationship between impulse, K, and τ and F ($K = \tau F/2$), and the fact that $F \propto A$, we have developed a magnitude scale M_k. This scale is dependent on the impulse and is tied to the initial explosive phase (first 110 s) of the May 18, 1980 Mt. St. Helens eruption whose magnitude is fixed at 4.6. We have also developed an equivalent magnitude scale, based on the wave amplitude at PPIG station. Table 1 gives a list of the explosions along with their magnitudes.

The largest recorded Popocatepetl explosion occurred on April 29, 1997 (M_k = 3.2). The impulse, $K = mv_o$, of this explosion (6.4×10^{11} N·s) was 220 times smaller than the value of 1.4×10^{14} N·s of the Mount St. Helens eruption computed from the 200 s source-time function given by *Kanamori et al.* [1984]. Assuming $v_o = 150$ m/s (Hugo Delgado, personal communication, 2001) as the ejecta velocity, we find that a mass of 4.3 x 10^9 kg was ejected in 9.6 s during the April 29, 1997 event. In contrast, the mass ejected during the initial 110 s of the Mt. St. Helens explosion may be estimated as ~2.2-6.6x10^{11} kg from the results of *Kanamori et al.* [1984] and *Brodsky et al.* [1999] which is 50 to 150 times more than during one of the largest Popocateptl explosions.

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