Effects of plate and slab viscosities on the geoid

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Abstract

The effects of plate rheology (strong plate interiors and weak plate margins) and stiff subducted lithosphere (slabs) on the geoid and plate motions, considered jointly, are examined with three-dimensional spherical models of mantle flow. Buoyancy forces are based on the internal distribution of subducted lithosphere estimated from the last 160 Ma of subduction history. While the ratio of the lower mantle/upper mantle viscosity has a strong effect on the long-wavelength geoid, as has been shown before, we find that plate rheology is also significant and that its inclusion yields a better geoid model while simultaneously reproducing basic features of observed plate motion. Slab viscosity can strongly affect the geoid, depending on whether a slab is coupled to the surface. In particular, deep, high-viscosity slabs beneath the northern Pacific that are disconnected from the surface as a result of subduction history produce significant long-wavelength geoid highs that differ from the observation. This suggests that slabs in the lower mantle may be not as stiff as predicted from a simple thermally activated rheology, if the slab model is accurate. © 1999 Elsevier Science B.V. All rights reserved.

Keywords: plates; rheology; subduction; lithosphere; viscosity; geoid; movement; plate tectonics; models

1. Introduction

Attempts to reproduce the observed global variations of the Earth’s geoid from models of flow in the mantle have so far not included small-scale lateral variations in rheology, of the kind associated with plate boundaries and subducted lithosphere, partly because of the difficulty of such numerical computations. Many studies have assumed horizontally uniform viscosity (e.g., [1–6]), while some more recent studies have included long-wavelength horizontal variations in viscosity [7–10]. The long-wavelength lateral viscosity variations do not affect the geoid significantly at degrees 2 and 3 [7,8]. Recent advances in computer power and computational methods have made more realistic computations possible. Moresi and Gurnis [11] have shown that slab viscosities can have important effects on the geoid in a regional model of a subduction zone. Simons [12] found that weak plate margins might also have important effects on the geoid in 2D numerical models. We show here with a 3D spherical shell model of the mantle that the influence of lateral variations in surface lithospheric viscosity on the geoid is more significant than previous studies [6] indicated. We also confirm that the effects of stiff subducted lithosphere are potentially large [11]. In addition, we find that the effects of slabs depend on the slabs’ topology.

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The difficulty of reproducing surface plate motions from mantle flow models that lack strong plates and weak plate boundaries has been recognized for some time [13–16]. The main long-wavelength features of the geoid can be reproduced reasonably well in simple flow models that have a laterally uniform lithosphere and a free-slip top surface, so long as the lithospheric viscosity is no more than 10–50 times the upper mantle viscosity [4]. Such models of course do not reproduce the surface plate motions at all well. This left the question of whether models that yielded more plate-like velocities would still match the geoid.

A step towards answering this question was taken by Ricard and Vigny [5], who introduced a torque-balancing method of including plate-like motions. In this method the real plate geometry is imposed, the plates have piecewise-constant velocities, and the value of each plate velocity is found by requiring the net torque on the plate from the computed flow to be zero. Using this formulation for plates, Thoraval and Richards [6] found that including plates did not significantly improve the fit to the observed geoid. The fit obtained was essentially the same as using a no-slip surface boundary condition, and neither was as good as using a free-slip boundary condition. Furthermore, the plate and no-slip models required the ‘lithosphere’ viscosity to be no greater than the interior mantle viscosity. Together, the results imply that mobility of the surface is important to obtaining a realistic geoid. Even with a no-slip surface, the viscosity of the lithosphere was required to be less than about 100 times the interior viscosity, although there is some trade-off with the viscosity of the lower mantle.

A common concern with the no-plate models is that the ‘lithosphere’ viscosity is required to be unrealistically low in order to maintain its mobility. There is also a concern that the torque-balance method ignores inter-plate forces at plate boundaries [5,9], and these might significantly affect results [9,17]. There is thus a need to try to include plate-like behavior in a more direct way in order to evaluate its influence on the geoid [9]. We have done this by prescribing high-viscosity plate interiors and low-viscosity plate margins. This method has been used successfully in two-dimensional models to obtain plate-like velocities [14–16].

We have also investigated the effect of making subducted lithospheric ‘slabs’ more viscous than the ambient mantle interior. This is expected because the slabs are colder than ambient mantle, and it is potentially important because the stiffness of slabs affects their coupling to the top and bottom surfaces of the mantle, and this can strongly influence the resulting geoid. This has been demonstrated in the regional subduction model of Moresi and Gurnis [11], but it has not been evaluated in a global context.

2. Models

We have computed the flow in three-dimensional spherical mantle models by a finite element method. The flow is driven by the negative buoyancy of slabs whose spatial distribution has been predicted from the past 160 Ma of subduction history [1–3]. Lateral viscosity variations are prescribed according the geometry of surface plates and subducted slabs, and the lower mantle is given a higher viscosity than the upper mantle. The models have a combination of higher resolution and greater viscosity contrast than previous models.

It is plausible to include only the negative buoyancy of subducted lithosphere to drive the flow, and to ignore positively buoyant upwellings, because it has been inferred that mantle flow is predominantly due to the top thermal boundary layer (the lithosphere), with only about 10% of the mantle heat budget being transported by mantle plumes rising from a bottom thermal boundary layer [18,19]. We have used the slab distribution model of Ricard et al. [1], who estimated the location of subducted lithosphere using plate reconstruction for the past 160 Ma and the assumption that slabs sink approximately vertically in the mantle at a rate depending on an inferred increase of viscosity with depth [1–3]. We will refer to this as the subduction history (SH) model.

An alternative way to estimate the buoyancy distribution in the present mantle is to convert seismic tomography structures into a density model. Recent high-resolution body wave tomography models are dominated by slab-like structures beneath subduction zones [20,21], and the SH model shows broad similarities to the tomography models. Thoraval and
Richards [6] have shown that the calculated geoid is not very sensitive to the choice of buoyancy model, comparing the SH model and two recent long-wavelength tomography models [22,23]. In this study we use a modified SH slab model [1] as input density model (Fig. 1a,b). The density anomalies in the original SH slab model were expressed as spherical harmonics up to degree 15 for...
the whole mantle [1]. We have replaced the upper mantle slabs of the SH model with higher-resolution and better-defined slabs, while the lower mantle slabs have been kept the same as in the original SH model. Because our finite element grid has a horizontal resolution of about 100 km throughout the outer surface, the upper mantle slabs are assumed to be 400 km wide with vertical dip angles. In order to conserve buoyancy for a 100-km-thick subducting lithosphere, density anomalies in these upper mantle slabs are reduced accordingly. The plate margins are assumed to be 400 km wide regardless of their tectonic style. In subduction zones, converging margins lie directly above the vertical slabs.

Two features of the SH slab model (Fig. 1a,b) have important effects on our results. (1) Slab density anomalies have greater power within the lower mantle than in the upper mantle. This is because the slabs in the lower mantle are assumed to thicken substantially due to an increased viscosity in the lower mantle [1]. (2) There are significant vertical gaps in some slabs. These gaps result from changes in plate motion directions in the plate reconstruction models (e.g., the change in the Pacific plate motion at 43 Ma) [1–3].

We compute models with progressively more realistic rheological structures for lithosphere and slabs to examine how the rheology influences the geoid and plate motion.

3. Formulation and numerical approach

Mantle flow is governed by the conservation equations of mass and momentum:

\[ \nabla \cdot \mathbf{u} = 0 \quad (1) \]

\[ -\nabla P + \nabla [\eta (\nabla \mathbf{u} + \mathbf{u} \nabla)] + \delta \rho \mathbf{g} = 0 \quad (2) \]

where \( \mathbf{u} \) is the velocity; \( P \) is the dynamic pressure; \( \delta \rho \) is the density anomaly; \( g \) is the gravitational acceleration; \( \eta \) is the viscosity; and the differential operators are in a spherical geometry. The top and core–mantle boundaries are subjected to free-slip boundary conditions.

For a spherically symmetric viscosity structure, Eqs. 1 and 2 can be solved analytically [17], but for an arbitrary viscosity structure, numerical approaches are necessary. Our numerical software CitcomS is built upon the 3D Cartesian finite element software Citcom [11] which uses a robust Uzawa algorithm for an incompressible medium [24]. Our grid provides relatively uniform resolution in both polar and equatorial regions, similar to that in [25]. We use eight-node trilinear elements that lead to accurate and stable pressure solutions for incompressible media [24]. We have implemented a full multi-grid solver with a consistent projection scheme in CitcomS.

Our finite element grid has a horizontal resolution of about 100 km throughout the outer surface. In the radial direction, grid spacings for the lithosphere, the upper and lower mantle are about 20 km, 40 km, and 70 km, respectively.

By comparing with analytic solutions for both isoviscous and layered viscosity models [17], we found that our numerical solutions from CitcomS are accurate with relative errors less than one percent for both velocity and stress (and hence for dynamic topography) for viscosity contrasts as large as \( 10^4 \). A detailed description of our numerical method and the benchmarks for both instantaneous Stokes’ flow and fully time-dependent thermal convection will be presented in a separate paper (Zhong et al., in prep.). For a given density structure \( \delta \rho \) and viscosity structure \( \eta \), flow velocity and dynamic topography at the bottom and top boundaries can be determined from Eqs. 1 and 2. Gravity anomalies and the geoid can be computed from dynamic topography and density anomalies \( \delta \rho \) [4]. In computing the geoid we take account of self-gravitation following the approach of Zhang and Christensen, in which the fluid core is treated as a self-gravitating body with no shear stress [7].

4. Results

4.1. Effects of plate rheology

In the first model (Case 1), we use a spherically symmetric viscosity structure inferred from the long-wavelength geoid by Hager and Richards [4]. In this four-layer viscosity model, the relative viscosities for the top 100-km, upper mantle, transition zone, and lower mantle are 3.0, 0.1, 3.0, and 30, respectively (Table 1). The model geoid resembles some of the
main features of the observed geoid (Fig. 1c) with geoid highs over subduction zones, although the amplitude of our model geoid is larger (Fig. 2a) and the central Pacific high is not as broad. There is a reasonably good correlation between the observed geoid and that of Case 1 for degrees 2 to 6, except degree 3 (Fig. 3). The overall fit to the geoid is expected because the viscosity structure in Case 1 is similar to that used by Ricard et al. [1] who reported a reasonable fit to the observed geoid with the SH slab model. Because Case 1 does not include realistic plate rheology, surface velocity shows zero vorticity and a distributed strain rate and divergence (Fig. 4a), which do not resemble the observations [26] (Fig. 1d–f).

We now demonstrate the influence of a strong lithosphere. Case 2 is identical to Case 1 except that lithospheric viscosity is increased by a factor of 100 (Table 1). This leads to a significantly different geoid which shows a large geoid high in the middle of the Pacific and geoid lows over the Americas and the Eurasian plate, where slabs exist in the lower mantle.
Table 1
Model parameters

<table>
<thead>
<tr>
<th>Case</th>
<th>(\eta_{\text{lith}})</th>
<th>(\eta_{\text{um}})</th>
<th>(\eta_{\text{trans}})</th>
<th>(\eta_{\text{lm}})</th>
<th>(R_{\text{slab/mantle}})</th>
<th>(R_{\text{margin/lith}})</th>
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<td>3.0</td>
<td>30</td>
<td>1</td>
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<tr>
<td>4</td>
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<td>60</td>
<td>1</td>
<td>1/300</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
<td>0.1</td>
<td>3.0</td>
<td>60</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

\(\eta_{\text{lith}}, \eta_{\text{um}}, \eta_{\text{trans}}, \text{and } \eta_{\text{lm}}\) are the viscosities for the lithosphere (0–100 km), upper mantle (100 km–410 km), transition zone (410 km–670 km), and lower mantle (670 km–2900 km), respectively.

\(R_{\text{slab/mantle}}\) is the ratio of slab viscosity to ambient mantle viscosity at the same depth; \(R_{\text{margin/lith}}\) is the ratio of plate margin viscosity to lithospheric viscosity.

(Fig. 2b). This is because the increased lithospheric viscosity in Case 2 enhances the coupling between the mantle buoyancy and the surface [4], which leads to a larger surface depression over slabs than in Case 1. The surface flow velocity is reduced by more than a factor of 10 in comparison with Case 1, although the flow pattern is similar (Fig. 4b).

In Case 3 we include weak plate margins. This is the only difference between Cases 2 and 3 (Table 1). The weak margins mobilize surface plate motion that now has a similar average velocity to that in Case 1 with a uniformly weak lithosphere (Fig. 4). The surface motion shows concentrated divergence and vorticity (Fig. 4c,d), resembling the observations (Fig. 1e,f). Since plate motion for the African plate is small in our model (Fig. 4c), we may compare our model with the observed plate motions in a hot spot reference frame (Fig. 1d). The Pacific, South American, and Australian plates are generally in good agreement with the observations. However, there are some significant differences between the model and observed plate motions. For example, the Eurasian and North American plates show too large converging velocities towards the Pacific plate in the model, and the Nazca plate shows a different direction of plate motion than the observation (Fig. 1d and Fig. 4c).

Weak margins have a significant influence on the geoid. The geoid lows over subduction zones are reduced and even become regional highs in Case 3 (Fig. 2c), compared to Case 2 (Fig. 2b). This result is consistent with that found in 2D calculations [12]. In comparison with Case 1, the inclusion of plate rheology improves the correlation with the observed geoid at degree 3, but the correlation at degree 2 is reduced (Fig. 3). We have found that the geoid in Case 3 remains largely unchanged when a wider plate margin (600 km) is used.

As has been seen in previous studies [6], we find that the viscosity contrast between the upper and lower mantles still exerts a strong influence on the geoid in our models with more plate-like behavior. This can be exploited to improve the fit to the observed geoid. In Case 4, the lower mantle viscosity is increased by a factor of 2, which is otherwise identical to Case 3 (Table 1). Such an increase is within the uncertainty of lower mantle viscosity derived from other constraints [27,28]. The geoid in Case 4 is similar to that in Case 1 (Fig. 2d). This is because the increased lower mantle viscosity reduces the coupling of the slab buoyancy to the surface, counteracting the influence of the weak plate margins. The degree correlation with the observed geoid at degrees 2 and 3 is higher for Case 4 than for Case 1 (Fig. 3).

These results indicate that the inclusion of plate rheology significantly affects the geoid. They show further that the fit to the observed geoid can be modestly improved by taking account of this sensitivity.

4.2. Effect of stiff subducted slabs: 3D model

We now investigate the influence of strong slabs on the geoid. In Case 5, slabs are assumed to be
ten times more viscous than the ambient mantle at the same depth (Table 1). In order to focus on the influence of slab rheology, we do not include weak margins in Case 5. While all of the upper mantle slabs are assigned a high viscosity, the high-viscosity slabs in the lower mantle are defined as only those regions with density anomalies that are greater than 30% of the maximum density anomaly at the same depth. High-viscosity slabs result in a dramatically different geoid (Fig. 2e). While the geoid over the western Pacific and South America remains depressed, a prominent geoid high emerges over the northern Pacific and extends into a large part of North America. The correlation with the observed geoid is significantly degraded at both degrees 2 and 4, compared with the cases without high viscosity for the slabs (Fig. 3).

To understand the geoid in Case 5, we notice that under the northern Pacific and North America the slabs in the SH model are concentrated in the deep part of the lower mantle and are disconnected from the shallow mantle. On the other hand the slabs beneath the western Pacific and the Eurasian plate are connected to the surface and have smaller amplitudes in the lower mantle [1] (Fig. 1a,b). This slab structure results from the change in the Pacific plate motion around 43 Ma in the plate reconstruction model that was used to build the slab model [2].

For a radially symmetric viscosity structure, geoid kernels are negative in the lower mantle for long-wavelengths. As a result, slabs in the lower mantle produce negative geoid at long-wavelengths [4], as observed in Case 2 (Fig. 2b). However, when slabs are more viscous than ambient mantle, long-wavelength geoid over the slabs can be either positive or negative, depending on their depth and morphology. For slabs near the bottom of the lower mantle, like those under the northern Pacific and North America, a higher viscosity will enhance the coupling of the slabs to the core–mantle boundary and reduce the
coupling to the surface. This may lead to positive geoid over these slabs. If slabs only exist from the upper mantle to the middle of the lower mantle, like those under the western Pacific and the Eurasian plate, slab viscosity may only have a small influence on geoid. This is because the distance between slabs and the core–mantle boundary is comparable to that between the slabs and the top surface, increasing slab viscosity will not preferentially increase the coupling of slabs to either surface.

4.3. Effect of stiff subducted slabs: 2D model

Calculations in simple 2D Cartesian models of slabs support this explanation (Fig. 5). The layered viscosity structure in these 2D slab models is the same as that in Cases 4 and 5. In the first two 2D models, a lower mantle slab, which is assumed vertical, is placed above the core–mantle boundary, and there is a gap between the lower mantle and upper mantle slabs (Fig. 5a). When the slabs have the same viscosity as their ambient mantle, above the slabs is a strong negative long-wavelength geoid (Fig. 5a). However, when the slab viscosities are increased by a factor of 10, long-wavelength geoid becomes positive (Fig. 5a). In the other two 2D models, the lower mantle slab is placed immediately under the upper mantle slab, but the slab only extends to the middle of the lower mantle (Fig. 5b). When no lateral variations in viscosity are present, the long-wavelength geoid is again negative. However, the geoid remains negative when the slabs are ten times more viscous (Fig. 5b). The results from the latter 2D model are consistent with a previous study by Moresi and Gurnis [11] who showed that high viscosity for upper mantle slabs depresses the geoid over the slabs.

5. Discussion and conclusions

Our models show that plate rheology (strong plate interiors and weak plate margins) can have a significant influence on the long-wavelength geoid, compared with models with a laterally uniform top layer (Fig. 2c, compared with Fig. 2a and b). Furthermore, the inclusion of plate behavior in this way can improve the fit to the observed geoid, particularly if a complementary adjustment to the lower mantle viscosity is made (Case 4, Fig. 2d and Fig. 3). This result is in contrast to those of Thoraval and Richards [6], who found that the inclusion of plate motions with a torque-balance formulation in a model with a laterally uniform top layer degraded the fit relative to a model with a free-slip boundary, even when the viscosity in the top layer was allowed to be unrealistically low.

![Diagram](image-url)
As expected, plate rheology yields much more realistic plate motions than uniform lithosphere models. Plate rheology in Case 3 gives rise to concentrated divergence and vorticity in the surface motion that resemble those observed (Fig. 4c,d). However, the Eurasian and North American plates move too rapidly towards subduction zones in the models. This is most likely because subduction in the models is essentially symmetric: there is nothing in the models to favor the subduction of one plate rather than the other. This contrasts with subduction in the Earth, in which the occurrence of subduction faults causes the subduction to be nearly completely asymmetric, with only one plate descending. Since the Eurasian and North American plates are not subducting at their Pacific margins, their motion towards those trenches is small.

This kind of result has previously led to the suggestion that continental plates are more strongly coupled to the deep mantle, possibly through deep continental roots [3,29,30]. However, this mechanism would be only possible if the continental roots extend deep enough to effectively couple the continents to the higher-viscosity transition zone and lower mantle. For example, it has been suggested that there might be cold downwellings extending to greater depths under continental roots [31], but recent seismic evidence does not support it [32]. It is clear, on the other hand, that the models still require more realistic treatment of faulted plate margins, so that they yield realistic one-sided subduction and pure strike-slip motion [33].

An important finding of this work is the large effect on the long-wavelength geoid from the combination of slab topology and slab viscosity. The sign of the geoid over high-viscosity slabs depends on whether deep slabs are connected to the top surface (Fig. 5). This effect does not occur in models with a radially symmetric viscosity [4]. If deep, high-viscosity slabs are not connected to the surface, the long-wavelength geoid over the slabs is strongly positive. This effect is seen most strongly in the northern Pacific when high viscosity is assigned to slabs (Fig. 2e), and it greatly degrades the fit to the geoid (Fig. 3). This north-Pacific slab distribution in the SH model results from subduction before 43 Ma, when the Pacific plate was inferred to be moving more northward, and earlier still when the Kula plate was subducting. In contrast, the slabs beneath the western Pacific and the Eurasian plate result from the westward motion of the Pacific plate since 43 Ma, and are concentrated at the upper and middle depths of the mantle [1].

The poor fit to the observed geoid with high-viscosity deep slabs is surprising. It suggests that slabs in the lower mantle may be no stronger than ambient mantle. This would require some other physical processes to offset the effects of temperature on slab viscosity. Riedel and Karato [34] have argued that grain size reduction as a result of the olivine-spinel phase change at a depth of 410 km can lead to substantial reduction in slab viscosity, although it is unclear whether the weakening due to grain size reduction is sufficient to offset the effect of temperature-dependent viscosity. An alternative to this conclusion would be that the distribution of buoyancy in the SH model is not accurate. This might be true if the history of subduction used in the SH model is not accurate. For example, it has been questioned whether the motion of the Pacific plate changed around 43 Ma [35,36]. In principle it would be useful to test the effect of stiff slabs using buoyancy derived from seismic tomography, but there are practical limitations in doing this at present. The still-limited resolution of tomography means that the precise boundaries of slabs are not well defined and may give rise inaccurate buoyancies and even to artificial gaps in slabs. Also it is clear that some of the structure in tomography models is due to compositional variation and anisotropy [21,37], and so they cannot be translated simply into buoyancy structure.

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