Homework 7
Physics 2130
Modern Physics
Due: Wednesday October 23, 2002

1. Taylor and Zafiratos, problem 6.8. In (a), in particular show that $v = ke^2/n\hbar$.

   (a) From the angular momentum quantization condition, $L = n\hbar$, we get $mvr = n\hbar$ and, therefore, $v = n\hbar/mr$. Because the radii of the stable Bohr orbits is $r = n^2 a_B$, this implies that $v = \hbar/(mna_B)$. Because $a_B = \hbar^2/ke^2m$, this can be rewritten:

$$v = ke^2/n\hbar. \tag{1}$$

   (b) By equation (1), clearly v is largest when n is smallest (i.e., for $n = 1$) and $v = ke^2/\hbar$. Thus,

$$\beta = \frac{v}{c} = \frac{ke^2}{\hbar c} = \frac{ke^2}{hc/2\pi} = \frac{1.44}{1240/2\pi} \approx 7.3 \times 10^{-3},$$

   where we used the results from problem #1. This is a small fraction of the speed of light, so relativistic effects can be ignored.

   (c)

$$\frac{1}{\alpha} = \frac{\hbar c}{ke^2} = \frac{1240/2\pi}{1.44} \approx 137 \quad \rightarrow \quad \alpha \approx \frac{1}{137}.$$

2. Taylor and Zafiratos, problem 6.10. A series is characterized by a particular value of n', the label of the lower level of transition. The longest wavelength (lowest photon energy) is given by the Rydberg formula with $n = n' + 1$, and the shortest wavelength (highest photon energy) is found by letting $n \to \infty$. For the four series in question ($n' = 1, 2, 3, 4$) we find:

Series	n'	λ_{min} (nm)	λ_{max} (nm)
Lyman	1	91	122
Balmer	2	365	656
Paschen	3	820	1875
Brackett	4	1458	4050

   In the Lyman series, all wavelengths are less than 400 nm, which means the whole series lies in the ultraviolet (UV). The Balmer series overlaps the visible and some UV, and the Paschen series is completely in the infrared (IR). The first three series do not overlap one another, but the long wavelengths of the Paschen series overlap the short wavelengths of the Brackett.

3. Taylor and Zafiratos, problem 6.16.

   (a) The $n = 1$ orbit for the pion orbiting a carbon nucleus ($Z = 6$) has radius:

$$r = \frac{\hbar^2}{Zke^2m_\pi} = \frac{a_B}{Zm_\pi/m_e} = \frac{5.29 \times 10^{-11} m}{6 \times 273} = 3.2 \times 10^{-14} \text{ m}, \tag{2}$$

where we used the fact that the Bohr radius is $a_B = \hbar^2/(ke^2 m_e)$.

(b) This orbit is much larger than the radius of the carbon nucleus, so it is possible.

(c) For the lead nucleus $Z = 82$, and placing this value of Z into equation (2) we get $r = 2.4 \times 10^{-15}$ m. This value is smaller than the radius that we're given for the lead nucleus (7×10^{-15} m), so the orbit of the electron would be within the lead nucleus, which is impossible.

4. Taylor and Zafiratos, problem 6.20.

(a) $E_n = -Z^2 E_R/n^2$ and the transition considered is $(n \to 1)$ so the energy of the photon produced by the electron going from level n to 1 is:

$$\frac{hc}{\lambda} = hf = E_{\text{ph}} = Z^2 E_R \left(1 - \frac{1}{n^2}\right)$$

Solving for wavelength we get:

$$\begin{aligned} \lambda &= \frac{hc}{Z^2 E_R} \frac{n^2}{n^2 - 1} = \frac{1240 \text{ eV nm}}{13.6 \text{ eV } Z^2} \frac{n^2}{n^2 - 1} \\ &= \frac{91.2 \text{ nm}}{Z^2} \frac{n^2}{n^2 - 1}. \end{aligned} \tag{3}$$

(b) For uranium the atomic number $Z = 92$, so 91.2 nm/Z$^2 \approx 1.08 \times 10^{-2}$ nm. The $K_\alpha, K_\beta, K_\gamma$ transitions are $2 \to 1, 3 \to 1, 4 \to 1$, respectively, so from equation (3) they have wavelengths:

$$\begin{aligned} \lambda_\alpha &= 1.08 \times 10^{-2} \frac{2^2}{2^2 - 1} = 1.08 \times 10^{-2} \left(\frac{4}{3}\right) = 1.44 \times 10^{-2} \text{ nm} \\ \lambda_\beta &= 1.08 \times 10^{-2} \frac{3^2}{3^2 - 1} = 1.08 \times 10^{-2} \left(\frac{9}{8}\right) = 1.21 \times 10^{-2} \text{ nm} \\ \lambda_\gamma &= 1.08 \times 10^{-2} \frac{4^2}{4^2 - 1} = 1.08 \times 10^{-2} \left(\frac{16}{15}\right) = 1.15 \times 10^{-2} \text{ nm} \end{aligned}$$

4. Taylor and Zafiratos, problem 7.8.

In general, you have to use relativity theory when the kinetic energy of a particle exceeds it rest mass energy. Here, $K = 2$ MeV, which exceeds the rest mass energy of an electron, $mc^2 = 0.511$ MeV. Therefore, we cannot use the nonrelativistic expression for kinetic energy ($K = mv^2/2 = p^2/2m = h^2/2m\lambda^2$), but rather must use the Pythagorean relation, $E^2 = (pc)^2 + (mc^2)^2$, where $E = mc^2 + K = 2.511$ MeV. Therefore,

$$pc = \sqrt{E^2 - (mc^2)^2} = \sqrt{2.511^2 - .511^2} \text{ MeV} = 2.46 \text{ MeV}.$$

Because $p = h/\lambda$, $\lambda = h/p$, so multiplying and dividing by c:

$$\lambda = \frac{hc}{pc} = \frac{1240 \text{ MeV fm}}{2.46 \text{ MeV}} = 504 \text{ fm}.$$