

Homework 5

Physics 2130

Modern Physics

Due: Wednesday October 9, 2002

1. Taylor and Zafiratos, problem 4.8.

(a) From $R = R_0 A^{1/3}$ with $R_0 \sim 1.1$ fm and with a periodic table in hand we find:

<i>atom</i>	<i>A</i>	<i>R</i>
He	4	1.75 fm
C	12	2.5 fm
Fe	56	4.2 fm
Pb	207	6.5 fm
LR	260	7.0 fm

(b) $V = (4/3)\pi R^3 = (4/3)\pi (R_0 A^{1/3})^3 = (4/3)\pi R_0^3 A \propto A$. The density of nucleons (mass/volume) in the nucleus is about the same inside all nuclei because both the mass and the volume of the nucleus go up linearly with the mass number A .

2. Taylor and Zafiratos, problem 4.21.

$$\begin{aligned}
 H &= \text{heat released} = (\# \text{ electrons}) \times (\text{kinetic energy of each electron}) \\
 &= \left(\frac{Q}{e}\right) \times \left(\frac{1}{2}mu^2\right) = \frac{Qmu^2}{2e}.
 \end{aligned} \tag{1}$$

From problem #4.20, done in class, $\theta = \ell e B / mu$. Multiplying this by equation (1), we can solve for the velocity u :

$$H\theta = \frac{Qmu^2}{2e} \frac{\ell e B}{mu} = \frac{1}{2}Q\ell B u \Rightarrow u = \frac{2H\theta}{Q\ell B}. \tag{2}$$

Finally, taking the result for m/e from problem #4.20, we can use equation (2) for u to find m/e in terms of the appropriate quantities:

$$\frac{m}{e} = \frac{\ell B}{\theta u} = \frac{Q\ell^2 B^2}{2H\theta^2}.$$

3. Taylor and Zafiratos, problem 4.23.

(a)

$$\begin{aligned}
 \frac{\ell}{t_d} + \frac{\ell}{t_u} &= v_d + v_u \stackrel{(4.47)}{=} \frac{Mg}{6\pi r\eta} + \frac{qE - Mg}{6\pi r\eta} = \frac{qE}{6\pi r\eta} \\
 \frac{1}{t_d} + \frac{1}{t_u} &= \frac{qE}{6\pi r\eta\ell}
 \end{aligned}$$

(c)

$$\begin{aligned}v_d &= \frac{\ell}{t_d} = \frac{2r^2 \rho g}{9\eta} \rightarrow r^2 = \frac{9\eta\ell}{2\rho g t_d} \\r &= \left(\frac{9\eta\ell}{2\rho g t_d} \right)^{1/2} = 6.94 \times 10^{-7} \text{ m},\end{aligned}$$

where the numerical result comes from substituting values for η , ℓ , and ρ from Table 4.8, $g = 9.8 \text{ m/s}^2$, and $t_d = 15.1 \text{ sec}$ is taken by averaging the values in Table 4.7. The oil droplets, therefore, are pretty small, about a micron in diameter.

4. Assume that the surfaces of the Sun and the North Star behave like blackbodies. We can get a good estimate of the temperature of the surface of each star by measuring the wavelength at which the radiation peaks, λ_m , which for the Sun is about 510 nm and for the North Star is about 350 nm.

(a) What the surface temperature of the Sun and the North Star?

By Wien's displacement law:

$$\begin{aligned}T &= \frac{2.898 \times 10^{-3} \text{ m} \cdot ^\circ K}{\lambda_m} = \frac{2.898 \times 10^6 \text{ nm} \cdot ^\circ K}{\lambda_m} \\T_{\text{Sun}} &= \frac{2.898 \times 10^6 \text{ nm} \cdot ^\circ K}{510 \text{ nm}} = 5682^\circ K \\T_{\text{NS}} &= \frac{2.898 \times 10^6 \text{ nm} \cdot ^\circ K}{350 \text{ nm}} = 8280^\circ K\end{aligned}$$

- (b) Using Stefan's law, determine the power radiated from each square meter of stellar surface.

$$\begin{aligned}R &= \sigma T^4 = (5.6703 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}) T^4 \\R_{\text{Sun}} &= (5.6703 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}) 5682^\circ K^4 = 5.90 \times 10^7 \text{ W/m}^2 \\R_{\text{NS}} &= (5.6703 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}) 8280^\circ K^4 = 2.67 \times 10^8 \text{ W/m}^2\end{aligned}$$

5. Taylor and Zafiratos, problem 5.6.

We're given that the total power $P_T = 60 \text{ W}$, the visible power $P_v = 3 \text{ W}$, and $\lambda = 550 \text{ nm}$. Recall that the energy per photon is $E_e = hf = hc/\lambda$ where $hc = 1240 \text{ eV}\cdot\text{nm}$.

(a)

$$\begin{aligned}N &= \# \text{ photons/sec} = \frac{P_v}{E_e} = \left(3 \text{ J/s} \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} \right) \frac{\lambda}{hc} \\&= 1.88 \times 10^{19} \text{ eV/s} \frac{550 \text{ nm}}{1240 \text{ eV} \cdot \text{nm}} = 8.3 \times 10^{18} \text{ s}^{-1}.\end{aligned}$$

(b) The photons that the light=bulb produces are distributed over a surface area with radius of 1 ft. The pupil intercepts a small fraction of these photons, that we wish to find. The surface area of a sphere of radius 1 ft is $A_{\text{sphere}} = 4\pi(12\text{in})^2 \approx 580\pi \text{ in}^2$. The area of a pupil of diameter 1mm is $A_{\text{pupil}} = \frac{\pi}{4}(1\text{mm})^2(1\text{in}/25.4\text{mm})^2 = 3.9 \times 10^{-4}\pi \text{ in}^2$. Therefore:

$$\frac{A_{\text{pupil}}}{A_{\text{sphere}}} = \frac{3.9 \times 10^{-4}\pi}{580\pi} = 6.7 \times 10^{-7}.$$

Therefore, the number of photons/s intercepted by the pupil is:

$$\begin{aligned} N_p &= \# \text{ photons entering pupil/sec} = N \frac{A_{\text{pupil}}}{A_{\text{sphere}}} = 8.3 \times 10^{18} s^{-1} (6.7 \times 10^{-7}) \\ &= 5.6 \times 10^{12} s^{-1}. \end{aligned}$$

(c) The minimum number of photons visible to the human eye is about $N_{\text{min}} = 10^2$, so the number that intersects the pupil, $N_p/N_{\text{min}} \approx 10^{12}/10^2 \approx 10^{10}$, is about 10 orders of magnitude higher than this minimum threshold.