

Homework 4

Physics 2130

Modern Physics

Due: Wednesday September 25, 2002

1. Taylor and Zafiratos, problem 3.27.

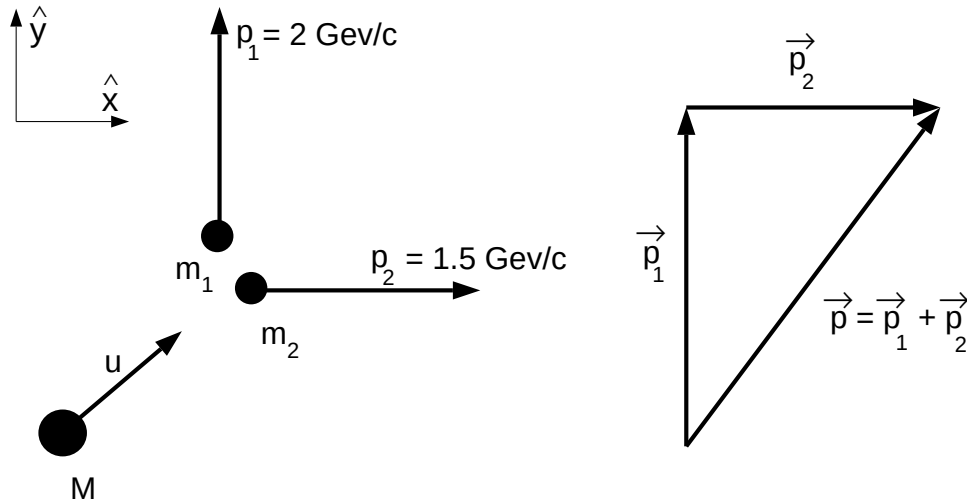


Figure 1:

We're told that a mass M decays into two particles with the following properties: $m_1 = 0.5 \text{ GeV}/c^2$, $m_2 = 1 \text{ GeV}/c^2$, $\vec{p}_1 = 2\hat{y} \text{ GeV}/c$, $\vec{p}_2 = 1.5\hat{x} \text{ GeV}/c$, where $\hat{x}$ and $\hat{y}$ are the unit vectors in the x and y directions, respectively.

The total energy of the particles, being a scalar, is found easily:

$$E_1 = \sqrt{(p_1 c)^2 + (m_1 c^2)^2} = \sqrt{2^2 + (0.5)^2} \text{ GeV} = 2.06 \text{ GeV},$$

$$E_2 = \sqrt{(p_2 c)^2 + (m_2 c^2)^2} = \sqrt{1.5^2 + 1^2} \text{ GeV} = 1.80 \text{ GeV}.$$

The total energy, therefore, is $E = 3.86 \text{ GeV}$.

To find the total momentum, a vector, consider Figure 1. $\vec{p} = \vec{p}_1 + \vec{p}_2$, so the magnitude of the final momentum $p = |\vec{p}| = |\vec{p}_1 + \vec{p}_2| = \sqrt{2^2 + 1.5^2} \text{ GeV}/c = 2.5 \text{ GeV}/c$. By conservation of momentum, this is also the momentum of mass M . Using the Pythagorean relation we can find mass M :

$$Mc^2 = \sqrt{E^2 - (pc)^2} = \sqrt{3.86^2 - 2.5^2} \text{ GeV} = 2.95 \text{ GeV}$$

$$M = 2.95 \text{ GeV}/c^2.$$

Using another useful formula we can find the initial speed of mass M :

$$\frac{u}{c} = \beta = \frac{pc}{E} = \frac{2.5}{3.6} = 0.65 \rightarrow u = 0.65c.$$

2. Taylor and Zafiratos, problem 3.30. This problem was done in class on Monday Sep 23, Lecture 11. The key in doing problems involving force is to use traditional SI units (kg, m, N, m/s, etc), rather than the energy units based on eV. Thus, we have to convert mass to kg and speed to m/s:

$$\begin{aligned} m &= 0.5 \text{ MeV}/c^2 = 9 \times 10^{-31} \text{ kg.} \\ u &= 0.7c = 2.1 \times 10^8 \text{ m/s.} \\ B &= 0.2 \text{ T} \\ q &= e = 1.6 \times 10^{-19} \text{ C.} \\ \gamma &= (1 - 0.49)^{-1/2} = 1.40 \end{aligned}$$

$$\begin{array}{ll} \text{Classical:} & R = \frac{mu}{eB} = 5.9 \text{ cm.} \\ \text{Relativity:} & R = \frac{\gamma mu}{eB} = 8.3 \text{ cm.} \end{array}$$

3. Taylor and Zafiratos, problem 3.33.

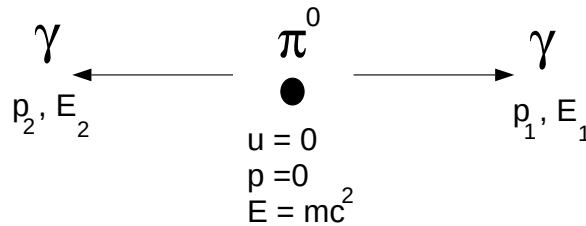


Figure 2:

By conservation of momentum: $0 = p_\pi = p_1 + p_2 \rightarrow p_1 = -p_2$. The photons have equal, but opposite momenta – i.e., they’re moving in opposite directions at the same speed as shown in Figure 2. Denote the absolute value of these momenta by p_γ . By conservation of energy: $m_\pi c^2 = E_\pi = E_1 + E_2 = |p_1|c + |p_2|c = p_\gamma c + p_\gamma c = 2p_\gamma c = 2E_\gamma \rightarrow E_\gamma = m_\pi c^2/2 = 67.5 \text{ MeV}$.

4. Taylor and Zafiratos, problem 3.40.

(a) Period is the distance traveled in one orbit, which is the circumference of a circle of radius R , divided by the speed of the particle:

$$T = \frac{2\pi R}{u} = \frac{2\pi}{u} \frac{p}{eB} = \frac{2\pi}{u} \frac{\gamma mu}{eB} = \frac{2\pi \gamma m}{eB}. \quad (1)$$

(b) The fractional increase in period is defined relative to the reference period, T_0 , which is the period of a non-relativistic particle with $\gamma = 1$. From equation (1), $T_0 = 2\pi m/eB$. Thus, the fractional increase in period is:

$$0.02 = \frac{T - T_0}{T_0} = \frac{\left(\frac{2\pi \gamma m}{eB} - \frac{2\pi m}{eB}\right)}{\frac{2\pi m}{eB}} = \gamma - 1 \equiv \frac{K}{mc^2} \rightarrow K = 0.02(mc^2) = 0.02(939 \text{ MeV}) = 20 \text{ MeV.},$$

where the equality before the arrow derives from the definition of kinetic energy, $K = (\gamma - 1)mc^2$.