Homework 2
Physics 2130
Modern Physics
Due: Wednesday September 11, 2002

1. Taylor and Zafiratos, problem 2.4.

   $v = 0.9$ c $\rightarrow \gamma = 3.2$. $\Delta t' = \Delta t/\gamma = 80$ y$/3.2 = 25$ y.

2. Taylor and Zafiratos, problem 2.9.

   $v = 0.99c \rightarrow \gamma = 7.09$. $t_{1/2}$(μ-on frame) $= 1.5$ μs. $t_{1/2}$(earth frame) $= \gamma t_{1/2}$(μ-on frame)$=$ $7.09(1.5) = 10.7$ μs. Time of transfit over 2000 m by the μ-on in the earth's atmosphere is $T = d/v =$
   2000m$/(.99)(3 \times 10^8) = 6.6$ μs. We find the number of half-lives in the earth's frame: $T/t_{1/2}$(earth frame) $= 6.6/10.7 = 0.63$. Therefore, the expected number of μ-ons observed at sea level will be: $N = N_0/2^{0.63} = 650/2^{0.63} = 420$ μ-ons at sea level.

   Note that if we erroneously ignored time dilation, then $T/t_{1/2}$(μ-on frame) $= 6.6/1.5 = 4.4$ μs, so $N = 650/2^{4.4} \approx 31$ μ-ons at sea level.

3. Taylor and Zafiratos, problem 2.15.

   $v = 0.8$ c $\rightarrow \gamma = 5/3, L' = 1$ m.

   (a) $L = L'/\gamma = 1/\gamma = 0.6$m.

   (b) $L = 1$ m.

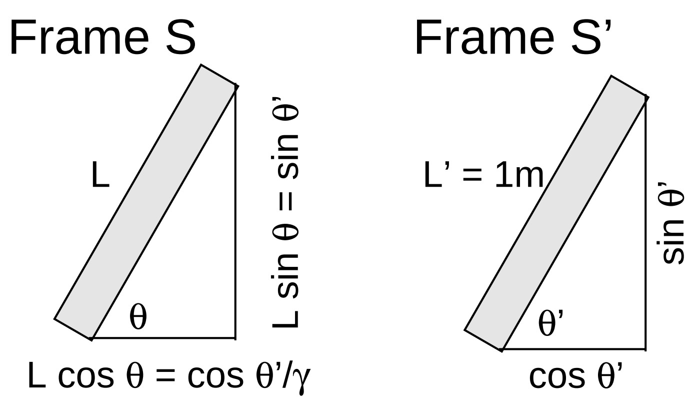

Figure 1: Tilted meter stick in stationary frame S and moving frame S'.

(c) See Figure 1. We're given that $\theta' = 60°$; $\sin\theta' = \sqrt{3}/2, \cos\theta' = 1/2, \tan\theta' = \sqrt{3}$. Thus, by the Phythagorean Theorem:

$$L = \left(\frac{\cos^2\theta'}{\gamma^2} + \sin^2\theta'\right)^{1/2} = 91.7\text{cm}.$$

We note in passing that $\theta = \arctan(\gamma\tan\theta') = 70.7°$.

(d) Now we're told that θ =60° in Figure 1. So $\sqrt{3}/4 = \tan\theta = \gamma \sin\theta' / \cos\theta' = \gamma \tan\theta'$. Thus, $\theta' = \arctan\left(\sqrt{3}/\gamma\right) = \arctan\left(3\sqrt{3}/5\right) = 46.1°$. Therefore, again we use the following equation but now set $\theta' = 46.1°$:

$$L = \left(\frac{\cos^2\theta'}{\gamma^2} + \sin^2\theta'\right)^{1/2} = 83.2\text{cm}.$$

4. Taylor and Zafiratos, problem 2.22.

We're given that $\Delta x = \Delta x' = 600$ m and that $\Delta t = 1$ μs. We wish to use the following Lorentz transformations with this information to find v and $\Delta t'$;

$$\begin{aligned} \Delta x' &= \gamma\left(\Delta x - v\Delta t\right) \\ \Delta x &= \gamma\left(\Delta x' + v\Delta t'\right) \end{aligned}$$

First, we find $\Delta t'$: $\gamma(\Delta x - v\Delta t) = \Delta x' = \Delta x = \gamma(\Delta x' + v\Delta t') = \gamma(\Delta x + v\Delta t')$. Canceling γ from both sides, then Δx and v, we see that $\Delta t' = -\Delta t = -1\mu$s.

Finding v is a little harder. We're given that: $\Delta x = \Delta x' = \gamma(\Delta x - v\Delta t)$. Thus, $(1-\gamma)\Delta x = -\gamma v \Delta t$ or:

$$\begin{aligned} 1 - \gamma^{-1} &= \frac{\gamma - 1}{\gamma} = v\frac{\Delta t}{\Delta x} \rightarrow \gamma^{-1} = 1 - v\frac{\Delta t}{\Delta x} \\ 1 - \frac{v^2}{c^2} &= 1 - \beta^2 = \gamma^{-2} = \left(1 - v\frac{\Delta t}{\Delta x}\right)^2 = 1 - 2v\frac{\Delta t}{\Delta x} + v^2\frac{\Delta t^2}{\Delta x^2} \\ v &= \left(\frac{2c\Delta t\Delta x}{c^2\Delta t^2 + \Delta x^2}\right)c = 0.8c. \end{aligned}$$

5. Taylor and Zafiratos, problem 2.26.

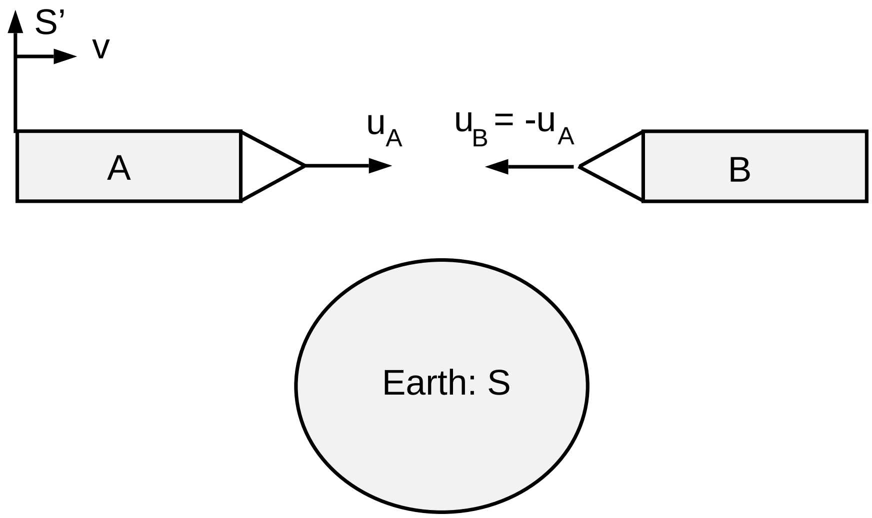

Figure 2: Two rockets in probme #5.

The physical setting is shown in Figure 2. In the earth's frame, the rockets are observed to be traveling toward each other with speeds of $u_A = 0.9c$ and $u_B = -u_A = -0.9c$, respectively. Rocket A's frame is moving at speed $v = 0.9c$ relative to the earth, and we find the speed of Rocket B from A's frame, u'_B, using the relativistic velocity addition formula:

$$u'_B = \frac{u_B - v}{1 - u_B v/c^2} = \frac{-0.9c - 0.9c}{1 + (0.9)^2} = \frac{-1.8c}{1.81} = -0.994c.$$