Homework 12
Physics 2130
Modern Physics
Due: Wednesday December 4, 2002

1. T&Z, #13.15.

(a)

$$\begin{aligned}
\#\text{ of } {}^{14}C \text{ atoms in 7 g of } C &= N_0 = (1.3\times10^{-12})\frac{7\text{ g}}{12\text{ g/mole}}(6.02\times10^{23}\text{ atoms/mole}) \\
&= 4.57\times10^{11}\text{ atoms of } {}^{14}C \\
\text{Decay constant } = r &= \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{5730\text{ y}} = 2.3\times10^{-10}\text{min}^{-1} \\
\text{Rate of decay} &= R_0 = rN_0 = 105\text{ decays/min}
\end{aligned}$$

(b) If n = # of half-lives:

$$\begin{aligned}
R &= 10\text{ decays/min} = \frac{R_0}{2^n} \\
2^n &= \frac{R_0}{R} \Rightarrow n\log 2 = \log\frac{R_0}{R} \Rightarrow n = \frac{\log(R_0/R)}{\log 2} = \frac{\log 10.5}{\log 2} = 3.39\text{ half-lives} = 3.39t_{1/2} \\
3.39t_{1/2} &= 3.39(5730\text{ years}) = 19,400\text{ years.}
\end{aligned}$$

2. β **Decay** T & Z #13.19. ${}^{14}C \rightarrow {}^{14}N + \bar{e} + \bar{\nu}$.

$$\begin{aligned}
K &= \Delta Mc^2 = \left[m_{atom}({}^{14}C) - m_{atom}({}^{14}N)\right]c^2 = [14.003242 - 14.003074]\,uc^2 \\
&= 0.000168(931.5\text{ MeV}/c^2)c^2 = 0.156\text{ MeV.}
\end{aligned}$$

3. **Radioactive Series.** T & Z #13.27.

For a radioactive decay to occur, the total mass after the decay must be less than the total mass of the parent objects. In other words, kinetic energy must be positive.

reaction	*comment*	*possible?*
${}^{228}Th \rightarrow {}^{227}Th + n$	$K = \Delta Mc^2 = (M_i - M_f)c^2$ = -7.13 MeV	No
$\rightarrow {}^{227}Ac + p$	$K = -6.39$ MeV	No
$\rightarrow {}^{228}Pa + \bar{\nu} + e^-$	$K = -2.10$ MeV	No
$\rightarrow {}^{224}Ra + \alpha$	$K = 5.50$ MeV	Yes
$\rightarrow {}^{216}Po + {}^{12}C$	$K = 24.99$ MeV	Yes

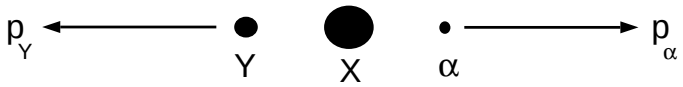

Figure 1: $X \to Y + \alpha$.

4. **Radioactive Series.** T & Z #13.34.

(a) By conservation of momentum, $|p_\alpha| = |p_Y| \equiv p$. Because we can treat these particles nonrelativistically, $K = p^2/2m$ which means that $K_\alpha = p_\alpha^2/2m_\alpha = p^2/2m_\alpha$ and $K_Y = p_Y^2/2m_Y = p^2/2m_Y$. Taking the ratio of kinetic energies we see:

$$\frac{K_\alpha}{K_Y} = \frac{m_Y}{m_\alpha} \to K_\alpha = \left(\frac{m_Y}{m_\alpha}\right) K_Y$$

(b) Typical $m_Y \approx 220$ u, $m_\alpha = 4 \to K_\alpha = 55K_Y = (55/56)K_{\text{total}} \approx 98\%K_{\text{total}}$.

(c) $X \to Y + e$. $m_e = 5 \times 10^{-4}$u, $m_Y = 1$u to 240u. Therefore, $K_e = (m_Y/m_e)K_Y \to 99.95\%$ to 99.9998% of K_{total}.

Actually these are maximum values in (b) and (c) because some of the kinetic energy will go to a neutrino.

5. T&Z #13.38.

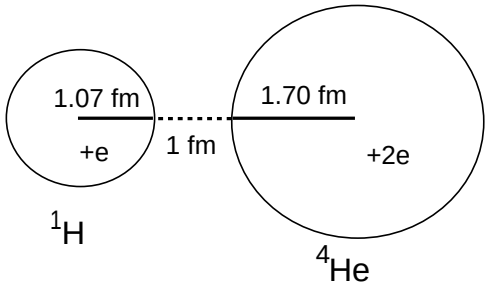

Figure 2:

(a) $R = R_0 a^{1/3} \to R(^1H) = 1.07$ fm, $R(^4He) = 1.70$ fm. d = distance between nuclear centers = 1 + 1.07 + 1.70 fm = 3.77 fm. Energy needed to bring one of the particles from infinity is just the potential energy:

$$E = \frac{kq_1q_2}{d} = \frac{2ke^2}{d} = \frac{2(1.44 \text{ MeV fm})}{3.77 \text{ fm}} = 0.76 \text{ MeV}.$$

(b) $R(^{238}U) = 6.63$ fm $\to d = 1 + 1.70 + 6.63 = 9.33$ fm.

$$E = \frac{kq_1q_2}{d} = \frac{2(92)ke^2}{d} = \frac{184(1.44 \text{ MeV fm})}{9.33 \text{ fm}} = 28 \text{ MeV}.$$

6. T & Z, #13.43.

(a) ${}^{207}_{84}Po \to {}^{207}_{85}At + e^- + \bar{\nu}$. Nucleon number and charge are conserved, but $\Delta K = (M_i + M_f)c^2 < 0$. Not possible.

(b) ${}^{213}_{86}Rn \rightarrow {}^{208}_{84}Po + {}^{4}_{2}He$. Nucleon number is not conserved. Not possible.

(c) ${}^{212}_{84}Po \rightarrow {}^{208}_{82}Pb + {}^{4}_{2}He$. Nucleon number and charge are conserved, and $\Delta K > 0$. Possible.

(d) ${}^{1}_{1}H + {}^{14}_{7}N \rightarrow {}^{12}_{6}C + {}^{4}_{2}He$. Nucleon number is not conserved. Not possible.

(e) $n + {}^{22}_{11}Na \rightarrow {}^{22}_{10}Ne + {}^{1}_{1}H$. Nucleon number and charge are conserved, and $\Delta K > 0$. Possible.