# Homework 11
# Physics 2130
# Modern Physics
# Due: Wednesday November 27, 2002

1.

(a) **T&Z, #12.8.** $R = R_0 A^{1/3}$ where $R_0 = 1.07$ fm.

$$\begin{array}{lll} A_{Al} = 27 & {}^{27}Al & R = 1.07\ (27)^{1/3} = 3.2\ \text{fm} \\ A_{Cu} = 63 & {}^{63}Al & R = 1.07\ (63)^{1/3} = 4.3\ \text{fm} \\ A_{Ur} = 238 & {}^{238}Al & R = 1.07\ (238)^{1/3} = 6.6\ \text{fm} \end{array}$$

(b) **T&Z, #12.9.** ${}^{7}Li$ with $E_{ph} = E_2 - E_1 = 0.48$ MeV.

$$\lambda = \frac{hc}{E_{ph}} = \frac{(1240\ \text{MeV fm})}{0.48\ \text{MeV}} = 2600\ \text{fm} = 2.6 \times 10^{-3}\ \text{nm}$$

If $E_{ph} = 1.8$ eV $\rightarrow \lambda = 690$ nm.

2. **T&Z #12.21.** ${}^{14}N$

(a) With $7p + 7N$ there is an even number of particles $\rightarrow$ total spin is an integer $= 0, 1, 2, 3, \ldots$.

(b) With $14p + 7\bar{e}$ there is an odd number of particles $\rightarrow$ total spin is an odd half-integer $= 1/2, 3/2, 5/2, \ldots$.

(c) Of course (a).

3. **T&Z #12.26.**

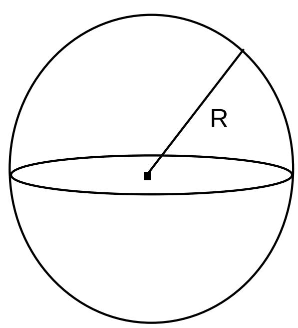


Figure 1: Sphere of total charge $Q$.

(a) This results from Gauss' Law, which states that if you're outside the charge-bearing region, then the electric field is equivalent to the field produces if all the charge was concentrated at the center of the charge-bearing region. Here the total charge is $Q$ located inside a sphere of radius $R$, so the electric field located at a point $r > R$ is just:

$$E(r) = \frac{kQ}{r^2} \qquad r \geq R \tag{1}$$

(b) If $r \geq R$:

$$\begin{aligned} \Delta V &= V(r_2) - V(r_1) = -\int_{r_1}^{r_2} E(r)dr \overset{(1)}{=} -kQ \int_{r_1}^{r_2} r^{-2}dr \\ &= -kQ(-r^{-1})\Big|_{r_1}^{r_2} = \frac{kQ}{r_2} - \frac{kQ}{r_1}. \end{aligned} \tag{2}$$

Letting $r_1 = \infty$ and $r_2 = r$, then $V(r_1) = V(\infty) = 0$ and

$$V(r_2) = V(r) = \frac{kQ}{r} \qquad r \geq R \tag{3}$$

(c) The total charge enclosed inside a sphere of radius $r \leq R$ is equal to the total charge multiplied by the ratio of the volumes:

$$Q' = Q\left(\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}\right) = Q\frac{r^3}{R^3} \tag{4}$$

Thus,

$$E(r) = \frac{kQ'}{r^2} = \frac{kQr}{R^3} \qquad r \leq R \tag{5}$$

Note that if $r = R$, both equations (1) and (5) yield

$$E(R) = \frac{kQ}{R^2}. \tag{6}$$

(d) If $r \leq R$:

$$\begin{aligned} \Delta V &= V(r_2) - V(r_1) = -\int_{r_1}^{r_2} E(r)dr \overset{(5)}{=} -\frac{kQ}{R^3}\int_{r_1}^{r_2} r dr = -\frac{kQ}{R^3}\left(\frac{1}{2}r^2\right)\Big|_{r_1}^{r_2} \\ &= \frac{kQ}{2R^3}\left(r_1^2 - r_2^2\right) \end{aligned} \tag{7}$$

If $r_1 = R$ and $r_2 = r$, then

$$\begin{aligned} V(r) - V(R) &= \frac{kQ}{2R^3}\left(R^2 - r^2\right) \\ V(r) &= V(R) + \frac{kQ}{2R^3}\left(R^2 - r^2\right) \overset{(3)}{=} \frac{kQ}{R} + \frac{kQ}{2R^3}\left(R^2 - r^2\right) \\ &= \frac{kQ}{2R}\left(3 - \frac{r^2}{R^2}\right) \qquad r \leq R \end{aligned} \tag{8}$$

Note that if $r = 0$,

$$V(0) \overset{(8)}{=} \frac{3kQ}{2R}. \tag{9}$$

4. **T&Z #12.23.** For $^{27}Al$, $Z = 13$ and $A = 27$. To offset the negative charge on the electron there would have to be an added proton, so the number of positive charges would be $Z+1$. The total charge to which the electron would be subjected would, therefore, be $Q = (Z+1)e$, where $e$ is the fundamental unit of charge. The radius of the nucleus would be $R = A^{1/3}R_0 = 3R_0$, where $R_0 = 1.07$ fm. Putting this all together:

$$U_e = -eV(0) \overset{(9)}{=} -\frac{3keQ}{2R} = \frac{3(Z+1)ke^2}{2(3R_0)} = \frac{3(14)1.44 \text{ MeV fm}}{6(1.07 \text{ fm})} = -9.4 \text{ MeV}.$$

Note that although this is negative, it is much too small to offset the minimum kinetic energy of about 180 MeV in equation (12.10) in the book. Thus, there's no holding an electron in the nucleus.

5. **T&Z #12.27.**

(a) $^{12}C$, so $Z = 6$ and $A = 12$. The proton would be subject to the electrostatic repulsion of the other $Z - 1 = 5$ protons. Thus, $Q = (Z-1)e = 5e$. In addition, $R = A^{1/3}R_0 = 12^{1/3}R_0$ where $R_0 = 1.07$ fm. From equation (9), $V(0) = 3kQ/2R = 3(5)ke/2(12^{1/3})R_0$, so

$$U(0) = eV(0) = \frac{3(5)ke^2}{2(12^{1/3})1.07 \text{ fm}} = 4.4 \text{ MeV}.$$

(b) $^{208}Pb$, so $Z = 82$ and $A = 208$.

$$U(0) = eV(0) = \frac{3(81)ke^2}{2(208^{1/3})1.07 \text{ fm}} = 28 \text{ MeV}.$$